1. Find all open intervals on which the function \( f(x) = \frac{x^2}{x^2 + 4} \) is decreasing.
   (a) (0, ∞)  
   (b) (−2, 2)  
   (c) (−∞, 0)  
   (d) (−∞, ∞)  
   (e) None of these

2. Find all critical numbers for the function \( f(x) = \frac{x - 1}{x + 3} \).
   (a) 1  
   (b) 1, −3  
   (c) −3  
   (d) 1, −1  
   (e) None of these

3. Find the values of \( x \) that give relative extrema for the function \( f(x) = 3x^5 - 5x^3 \).
   (a) Relative maximum: \( x = 0 \); Relative minimum: \( x = \sqrt{5}/3 \)
   (b) Relative maximum: \( x = -1 \); Relative minimum: \( x = 1 \)
   (c) Relative maxima: \( x = ±1 \); Relative minimum: \( x = 0 \)
   (d) Relative maximum: \( x = 0 \); Relative minima: \( x = ±1 \)
   (e) None of these

4. Find all intervals on which the graph of the function is concave upward: \( f(x) = \frac{x^2 + 1}{x^2} \).
   (a) (−∞, ∞)  
   (b) (−∞, −1) and (1, ∞)  
   (c) (−∞, 0) and (0, ∞)  
   (d) (1, ∞)  
   (e) None of these

5. Let \( f''(x) = 4x^3 - 2x \) and let \( f(x) \) have critical numbers −1, 0, and 1. Use the Second Derivative Test to determine if any of the critical numbers gives a relative maximum.
   (a) −1  
   (b) 0  
   (c) 1  
   (d) −1 and 1  
   (e) None of these

6. Find the limit: \( \lim_{x \to \infty} \frac{2x^4 + 6x^2 + 5}{3 + x^3} \).
   (a) \( \frac{2}{3} \)  
   (b) \( \infty \)  
   (c) 1  
   (d) 2  
   (e) None of these

7. Which of the following functions has a horizontal asymptote at \( y = 2? \)
   (a) \( \frac{x - 2}{3x - 5} \)  
   (b) \( \frac{2x}{\sqrt{x} - 2} \)  
   (c) \( \frac{2x^2 - 6x + 1}{1 + x^2} \)  
   (d) \( \frac{2x - 1}{x^2 + 1} \)  
   (e) None of these
8. Find the limit: \( \lim_{x \to -\infty} \left( x + \sqrt{x^2 + 3x} \right) \).
   (a) \(-\infty\) (b) \(-\frac{3}{2}\) (c) 0
   (d) \(-3\) (e) None of these

9. Find all points of inflection: \( f(x) = \frac{1}{12}x^4 - 2x^2 + 15 \).
   (a) (2, 0) (b) (2, 0), (−2, 0) (c) (0, 15)
   (d) \(\left(2, \frac{22}{3}\right), \left(-2, \frac{22}{3}\right)\) (e) None of these

10. The management of a large store wishes to add a fenced-in rectangular storage yard of 20,000 square feet, using the building as one side of the yard. Find the minimum amount of fencing that must be used to enclose the remaining 3 sides of the yard.
   (a) 400 ft (b) 200 ft (c) 20,000 ft
   (d) 500 ft (e) None of these

11. Use the graph of \( f' \) given in the figure to choose the true statement about \( f \).
   (a) \( f \) is decreasing on the interval (0, \( \infty \)).
   (b) \( f \) has a relative maximum at \( x = 0 \).
   (c) \( f \) is increasing on the interval (−1, 2).
   (d) \( f \) has a relative minimum at \( x = 2 \).
   (e) None of these

12. State why Rolle’s Theorem does not apply to the function \( f(x) = \frac{2}{(x + 1)^2} \) on the interval [−2, 0].
   (a) \( f \) is not continuous on [−2, 0]. (b) \( f(-2) \neq f(0) \)
   (c) \( f \) is not differentiable at \( x = -1 \). (d) Both a and c.
   (e) None of these

13. Find all extrema in the interval [0, 2\( \pi \)] if \( y = x + \sin x \).
   (a) \( \left(-1, -1 + \frac{3\pi}{2}\right), (0, 0) \)
   (b) \( (2\pi, 2\pi), (0, 0) \)
   (c) \( (2\pi, 2\pi), (\pi, \pi) \)
   (d) \( (\pi, \pi), (0, 0) \)
   (e) None of these

14. The side of a cube is measured to be 3.0 inches. If the measurement is correct to within 0.01 inch, use differentials to estimate the propagated error in the volume of the cube.
   (a) \( \pm 0.000001 \text{ in.}^3 \) (b) \( \pm 0.06 \text{ in.}^3 \) (c) \( \pm 0.027 \text{ in.}^3 \)
   (d) \( \pm 0.27 \text{ in.}^3 \) (e) None of these
1. Find all open intervals on which the function \( f(x) = \frac{x}{x^2 + x - 2} \) is decreasing.
   (a) \((−∞, ∞)\)  
   (b) \((−∞, 0)\)  
   (c) \((−∞, −2)\) and \((1, ∞)\)  
   (d) \((−∞, −2), (−2, 1)\) and \((1, ∞)\)  
   (e) None of these

2. Find all critical numbers for the function \( f(x) = (9 − x^2)^{3/5} \).
   (a) 0  
   (b) 3  
   (c) −3, 3  
   (d) −3, 0, 3  
   (e) None of these

3. Find the values of \( x \) that give relative extrema for the function \( f(x) = (x + 1)^2(x − 2) \).
   (a) Relative maximum: \( x = −1 \); relative minimum: \( x = 1 \)  
   (b) Relative maxima: \( x = 1, x = 3 \); Relative minimum: \( x = −1 \)  
   (c) Relative minimum: \( x = 2 \)  
   (d) Relative maximum: \( x = −1 \); Relative minimum: \( x = 2 \)  
   (e) None of these

4. Find all intervals on which the graph of the function is concave upward: \( f(x) = \frac{x − 1}{x + 3} \).
   (a) \((−∞, ∞)\)  
   (b) \((−∞, −3)\)  
   (c) \((1, ∞)\)  
   (d) \((−3, ∞)\)  
   (e) None of these

5. Let \( f''(x) = 3x^2 − 4 \) and let \( f(x) \) have critical numbers \(-2, 0, \) and \( 2 \). Use the Second Derivative Test to determine which critical numbers, if any, gives a relative maximum.
   (a) −2  
   (b) 2  
   (c) 0  
   (d) −2 and 2  
   (e) None of these

6. Find the limit: \( \lim_{x \to ∞} \frac{\sqrt{4x^2 − 1}}{x^2} \).
   (a) 4  
   (b) 0  
   (c) 2  
   (d) ∞  
   (e) None of these

7. Which of the following functions has a horizontal asymptote at \( y = −\frac{1}{2} \)?
   (a) \( \frac{x^3}{1 − 2x^3} \)  
   (b) \( \frac{x}{\sqrt{2x + 1}} \)  
   (c) \( \frac{2x^2 − 6x + 1}{1 + x^2} \)  
   (d) \( \frac{x − 1}{2x^2 + 1} \)  
   (e) None of these
8. Find the limit: \( \lim_{x \to \infty} \left( 2x - \sqrt{4x^2 + x} \right) \).
   (a) \(-\frac{1}{4}\)  (b) \(\infty\)  (c) \(-\frac{1}{2}\)  (d) 0  (e) None of these

9. Find all points of inflection: \( f(x) = x^3 - 12x \).
   (a) \((0, 0), \left( \pm \sqrt{12}, 0 \right)\)
   (b) \((0, 0)\)
   (c) \((2, 0), (-2, 0)\)
   (d) \((2, -16), (-2, 16)\)
   (e) None of these

10. A farmer has 160 feet of fencing to enclose 2 adjacent rectangular pig pens. What dimensions should be used so that the enclosed area will be a maximum?
   (a) \(4\sqrt{15} \text{ ft by } \frac{8}{3}\sqrt{15} \text{ ft}\)
   (b) \(40 \text{ ft by } \frac{80}{3} \text{ ft}\)
   (c) \(20 \text{ ft by } \frac{80}{3} \text{ ft}\)
   (d) \(40 \text{ ft by } 40 \text{ ft}\)
   (e) None of these

11. State why the Mean Value Theorem does not apply to the function \( f(x) = \frac{2}{(x + 1)^2} \) on the interval \([-3, 0] \).
   (a) \( f(-3) \neq f(0) \)
   (b) \( f \) is not continuous at \( x = -1 \).
   (c) \( f \) is not defined at \( x = -3 \) and \( x = 0 \).
   (d) Both (a) and (b)
   (e) None of these

12. Use the graph of \( f' \) given in the figure to choose the true statement about \( f \).
   (a) \( f \) has no relative extrema.
   (b) \( f \) is increasing on the interval \((-\infty, \infty)\).
   (c) \( f \) is decreasing on the interval \((-\infty, 1)\).
   (d) \( f \) has a relative maximum at \( x = 1 \).
   (e) None of these

13. Find all extrema in the interval \([0, 2\pi]\) for \( y = x - \cos x \).
   (a) \( \left( \frac{3\pi}{2}, \frac{3\pi}{2} \right), (2\pi, 2\pi - 1) \)
   (b) \((\pi, \pi + 1), (0, 0)\)
   (c) \((2\pi, 2\pi - 1), (0, 0)\)
   (d) \((2\pi, 2\pi - 1), (\pi, \pi + 1)\)
   (e) None of these

14. The radius of a sphere is measured to be 3.0 inches. If the measurement is correct to within 0.01 inch, use differentials to estimate the propagated error in the volume of the sphere.
   (a) \( \pm 0.000001 \text{ in.}^3 \)
   (b) \( \pm 0.36\pi \text{ in.}^3 \)
   (c) \( \pm 0.036\pi \text{ in.}^3 \)
   (d) \( \pm 0.06 \text{ in.}^3 \)
   (e) None of these
A graphing calculator is needed for some problems.

1. Determine from the graph whether \( f \) possesses extrema on the interval \((a, b)\).
   (a) Maximum at minimum at \( x = c \)
   (b) Maximum at no minimum
   (c) No maximum, minimum at \( x = b \)
   (d) No extrema
   (e) None of these

2. Use a graphing calculator to graph \( f(x) = x^{2/3} \). State why Rolle’s Theorem does not apply to \( f \) on the interval \([-1, 1]\).
   (a) \( f \) is not continuous on \([-1, 1]\).
   (b) \( f \) is not defined on the entire interval.
   (c) \( f \) is not differentiable at \( x = 0 \).
   (d) \( f(-1) \neq f(1) \)
   (e) None of these

3. Use the graph to identify the open intervals where the function is increasing or decreasing.
   (a) Increasing \((-\infty, 1]\) and \((-\infty, -3]\); decreasing \([1, \infty)\)
   (b) Increasing \((0, 2]\); decreasing \((-\infty, 0]\) and \([2, \infty)\)
   (c) Increasing \((-\infty, \infty)\)
   (d) Increasing \((-\infty, 0]\) and \((2, \infty]\); decreasing \([0, 2]\)
   (e) None of these

4. Let \( f(x) = (x + 2)^3 - 4 \). The point \((-2, -4)\) is ________________.
   (a) An absolute maximum
   (b) An absolute minimum
   (c) A critical point but not an extremum
   (d) Not a critical point
   (e) None of these

5. Given that \( f(x) = -x^2 + 12x - 28 \) has a relative maximum at \( x = 6 \), choose the correct statement.
   (a) \( f' \) is negative on the interval \((-\infty, 6)\)
   (b) \( f' \) is positive on the interval \((-\infty, \infty)\)
   (c) \( f' \) is negative on the interval \((6, \infty)\)
   (d) \( f' \) is positive on the interval \((6, \infty)\)
   (e) None of these
6. Use a graphing calculator to graph \( f(x) = \frac{1}{(x + 1)^2} \). Use the graph to determine the open intervals where the graph of the function is concave upward or concave downward.

(a) Concave downward: \((-\infty, \infty)\)
(b) Concave downward: \((-\infty, -1)\); concave upward: \((-1, \infty)\)
(c) Concave downward: \((-\infty, -1)\) and \((-1, \infty)\)
(d) Concave upward: \((-\infty, -1)\) and \((-1, \infty)\)
(e) None of these

7. The figure given in the graph is the second derivative of a polynomial function, \( f \). Choose a graph of \( f \).

(e) None of these

8. Let \( f(x) \) be a polynomial function such that \( f(-2) = 5 \), \( f'(-2) = 0 \), and \( f''(-2) = 3 \).

(a) Relative maximum
(b) Relative minimum
(c) Intercept
(d) Point of inflection
(e) None of these

9. Use a graphing calculator to graph \( f(x) = \frac{2x + 5}{1 - x} \). Use the graph to find the limit: \( \lim_{x \to \infty} \frac{2x + 5}{1 - x} \)

(a) 0
(b) \( \infty \)
(c) 2
(d) 5
(e) None of these
10. Find the limit: \( \lim_{x \to \infty} \frac{a - bx^4}{cx^4 + x^2} \).
   (a) 0  
   (b) \( \infty \)  
   (c) \(-\frac{b}{c}\)  
   (d) \( \frac{a}{c} \)  
   (e) None of these

11. Consider \( f(x) = \frac{x^2}{x^2 + a}, a > 0 \). Use a graphing calculator to determine the effect on the graph of \( f \) if \( a \) is varied.
   (a) Each \( y \) value is multiplied by \( a \).
   (b) As \( a \) increases, the vertical tangent lines move further from the origin.
   (c) The graph of the curve is shifted \( \sqrt{a} \) units to the left.
   (d) As \( a \) increases, the curve approaches its asymptotes more slowly.
   (e) None of these

12. Determine the function whose graph has vertical asymptotes at \( x = \pm 2 \) and a horizontal asymptote at \( y = 0 \).
   (a) \( f(x) = \frac{x}{(x - 2)^2} \)  
   (b) \( f(x) = \frac{x}{x^2 + 4} \)  
   (c) \( f(x) = \frac{3x}{x^2 - 4} \)  
   (d) \( f(x) = x(x - 2)(x + 2) \)  
   (e) None of these

13. Find the point on the graph of \( y = \sqrt{x + 1} \) closest to the point \( (3, 0) \).
   (a) \( (0, 1) \)  
   (b) \( \left( \frac{5}{2}, \sqrt{\frac{7}{2}} \right) \)  
   (c) \( (3, 2) \)  
   (d) \( (2, \sqrt{3}) \)  
   (e) None of these

14. Use Newton’s Method to approximate the real zero of the function in the interval \([-1, 0]\): \( f(x) = x^3 + x + 1 \).
   (a) \(-0.83\)  
   (b) \(-0.68\)  
   (c) \(-0.48\)  
   (d) \(-0.23\)  
   (e) None of these

15. Find \( dy \) for \( y = \sec 3x \).
   (a) \( \sec 3x \, dx \)  
   (b) \( \sec 3x \tan 3x \, dx \)  
   (c) \( 3 \sec^2 3x \, dx \)  
   (d) \( 3 \tan^2 3x \)  
   (e) None of these

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1. Find the function \( f \) that has the derivative \( f'(x) = 4x + 1 \) and whose graph passes through the point \( (1, 0) \).

2. Determine the intervals where \( f \) is increasing or decreasing for \( f(x) = \frac{1}{x^2} \).

3. Find all critical numbers: \( f(x) = x\sqrt{2x + 1} \).

4. Find extrema: \( y = \frac{2x}{(x + 4)^2} \).

5. Find all extrema in the interval \([0, 2\pi]\) if \( y = \sin x + \cos x \).

6. Find all intervals for which the graph of the function \( y = 8x^3 - 2x^4 \) is concave downward.

7. Let \( f(x) = x^3 - x^2 + 3 \). Use the Second Derivative Test to determine which critical numbers, if any, give relative extrema.

8. Find the limit: \( \lim_{x \to \infty} \frac{\sqrt{4x^2 - 1}}{x} \).

9. Use the techniques learned in this chapter to sketch the graph of \( y = \frac{3}{(1 - x)^2} \).

10. Use the techniques learned in this chapter to sketch the graph of \( y = x^3 - 3x + 1 \).

11. Find all points of inflection for the graph of the function \( f(x) = 2x(x - 4)^3 \).

12. The management of a large store has 1600 feet of fencing to fence in a rectangular storage yard using the building as one side of the yard. If the fencing is used for the remaining 3 sides, find the area of the largest possible yard.

13. Calculate 3 iterations of Newton’s Method to approximate a zero of \( f(x) = x^3 - x + 1 \). Use \( x_0 = -1.5000 \) as the initial guess and round to 4 decimal places after each iteration.

14. Find all horizontal asymptotes: \( f(x) = \frac{2}{x - 3} - \frac{x}{x + 2} \).

15. The volume of a cube is claimed to be 27 cubic inches, correct to within 0.027 cubic inches. Use differentials to estimate the propagated error in the measurement of the side of the cube.
A graphing calculator is needed for some problems.

1. Find the function $f$ that has the derivative $f'(x) = 4x - 1$ and whose graph passes through the point $(1, 0)$.

2. Determine from the graph whether $f$ possesses extrema on the interval $(a, b]$.

3. Consider $f(x) = \frac{1}{(x - 3)^2}$.
   a. Sketch the graph of $f(x)$.
   b. Calculate $f(2)$ and $f(4)$.
   c. State why Rolle’s Theorem does not apply to $f$ on the interval $[2, 4]$.

4. Consider $f(x) = \sqrt{x}$. Find all values, $c$, in the interval $[0, 1]$ such that the slope of the tangent line to the graph of $f$ at $c$ is parallel to the secant line through the points $(0, f(0))$ and $(1, f(1))$.

5. Use a graphing calculator to graph the function $f(x) = 9x^4 + 4x^3 - 36x^2 - 24x$.
   a. Adjust the viewing window and use the zoom and trace features to estimate the $x$-values of the relative extrema to one decimal place.
   b. Use calculus to find the actual $x$-values of the relative extrema.

6. Use a graphing calculator to graph the function $f(x) = x^3 + 3x^2 - 9x - 10$. Then use the graph to determine open intervals of increasing or decreasing.

7. A differentiable function $f$ has only one critical number: $x = -3$. Identify the relative extrema of $f$ at $(-3, f(-3))$ if $f'(-4) = \frac{1}{2}$ and $f'(-2) = -1$.

8. Use a graphing calculator to graph $f(x) = \frac{1}{x - 2} + 1$. Use the graph to determine the open intervals where the graph of the function is concave upward or concave downward.
9. The graph of a polynomial function, $f$, is given. On the same coordinate axes sketch $f'$ and $f''$.

![Graph](image1.png)

10. Find the limits:

   a. \[ \lim_{x \to +\infty} \frac{2x}{\sqrt{x^2 + 1}} \]

   b. \[ \lim_{x \to -\infty} \frac{2x}{\sqrt{x^2 + 1}} \]

   c. \[ \lim_{x \to \infty} \frac{2x}{\sqrt{x^2 + 1}} \]

11. Consider \( f(x) = \frac{x^2 - 4x + 1}{x^2 + 1} \).

   a. Find all asymptotes.

   b. Use a graphing calculator to graph \( f \).

   c. Use the graph to find the point on the graph where the curve crosses the horizontal asymptote.

12. Create a function whose graph has a vertical asymptote at \( x = 0 \) and a slant asymptote at \( y = x - 1 \).

13. An open box is to be made from a rectangular piece of cardboard, 7 inches by 3 inches, by cutting equal squares from each corner and turning up the sides.

   a. Write the volume, \( V \), as a function of the edge of the square, \( x \), cut from each corner.

   b. Use a graphing calculator to graph the function, \( V \). Then use the graph of the function to estimate the size of the square that should be cut from each corner and the volume of the largest such box.

14. Use a graphing calculator to graph \( f(x) = 0.2x^3 + 3.1x - 1.4 \).

   a. Use the graph to estimate (to one decimal place) the real zero of \( f \).

   b. Approximate this zero using the value found in part a and Newton’s Method until two successive approximations differ by less than 0.001.

15. Consider \( f(x) = x^3 \).

   a. Find an equation of the tangent line, \( T \), at the point \( (2, 8) \).

   b. Graph \( f \) and \( T \) on the same coordinate axes using a graphing calculator.

   c. Use the graphs to estimate \( f(2.1) \) and \( T(2.1) \).

   d. Calculate the actual values of \( f(2.1) \) and \( T(2.1) \).