Calculus

Grade 12

Prerequisite: Pre-Calculus

Credit Value: 5

ABSTRACT

Calculus is designed for the student who will be taking calculus in college, but is not necessarily seeking advanced placement credit. The course builds upon concepts from Pre-Calculus, such as inverse functions, exponential functions, logarithms, and limits, and applies them to a detailed study of differentiation and integration. Considerable time is spent on the applications of calculus to the fields of business, finance, and physics through the study of related rates, optimization, and analysis of curves and slopes. Students are also required to justify answers and conclusions using graphing calculators or Microsoft Excel spreadsheets and then effectively communicate their reasoning orally and in writing. Calculus provides students with the conceptual framework and analytical skills necessary for application to real-world problems and to be successful in the college calculus course.

Adopted by the Somerville Board of Education on July 25, 2017
### Calculus – Grade 12

<table>
<thead>
<tr>
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<th>September</th>
<th>October</th>
<th>November</th>
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#### Essential Question:
- How do the characteristics of functions connect to the traits of their graphs?
- How do limits serve as a means to better understand functions and their behavior?
- How is the concept of derivative representative of instantaneous rate of change?
- How do exponential functions and logarithms apply to growth and decay?
- When is implicit differentiation necessary?

#### Content:
- Properties of Functions and Their Behavior
- Limits of Functions
- Derivatives
- Derivatives of Exponential and Logarithmic Functions
- Implicit Differentiation and Related Rates

#### Skills and Topics:
- Interpret function and composite function notation
- Identify the domain and range of functions
- Graph functions
- Analyze trends among types of functions, their domains and ranges, and their graphs
- Justify conclusions about functions and their graphs using a graphing calculator
- Apply knowledge of functions to problem solving
- Understand the concept of limit and its purpose
- Calculate the limit of functions
- Apply the properties of limits to aid in problem solving
- Formulate a definition of continuity using limits
- Determine the location of asymptotes using limits
- Compare and contrast average rate of change and instantaneous rate of change
- Develop a definition of a derivative by connecting limits with instantaneous rate of change
- Calculate the derivative of a function using the limit definition
- Identify intervals on which a function is or is not differentiable
- Apply the power, chain, product, and quotient rules to find derivatives
- Perform algebraic manipulation on expressions to aid in the evaluation of derivatives
- Find the equation of the tangent line to a curve at a given point
- Integrate the concepts of position, velocity, and acceleration into higher-order derivatives
- Calculate the derivative of natural exponential and logarithmic functions
- Calculate the derivative of exponential functions with bases other than e (the base of the natural logarithm)
- Apply skills to problem solving involving exponential and logarithmic functions (e.g., growth/decay, sound waves)
- Identify equations that require implicit differentiation
- Modify rules for finding derivatives in order to differentiate implicitly
- Apply implicit differentiation to solving related rate problems
- Compare and contrast effective and ineffective methods for solving related rate problems
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| Interdisciplinary Connections: | *ELA: RST.9-10.1-9, WHST.9-10.1, WHST.9-10.10  
World Language: 7.1.AL.A.3  
21st Century Life/Careers: CRP2, CRP8, CRP9, CRP12  
| 21st Century Themes: | ☒ Global Awareness  ☐ Civic Literacy  ☐ Financial, Economic, Business, and Entrepreneurial Literacy  ☐ Health Literacy |                                                                                         |                                                                                           |                                                                                           |                                                                                           |
| 21st Century Skills: | ☒ Creativity and Innovation  ☐ Media Literacy  ☒ Critical Thinking and Problem Solving  ☐ Life and Career Skills  ☐ Information and Communication Technologies Literacy  ☐ Communication and Collaboration  ☐ Information Literacy |                                                                                         |                                                                                           |                                                                                           |                                                                                           |
| Careers:             | Applicable career options are discussed as they arise throughout the mathematics program. Career options include, but are not limited to, the following career clusters: Agriculture, Food, and Natural Resources Career Cluster; Architecture and Construction Career Cluster; Arts, A/V Technology, and Communications Career Cluster; Business, Management, and Administration Career Cluster; Education and Training Career Cluster; Finance Career Cluster; Government and Public Administration Career Cluster; Health Science Career Cluster; Hospitality and Tourism Career Cluster; Human Services Career Cluster; Information Technology Career Cluster; Law, Public Safety, Correction, and Security Career Cluster; Manufacturing Career Cluster; Marketing Career Cluster; Science, Technology, Engineering and Mathematics Career Cluster; Transportation, Distribution, and Logistics Career Cluster. |                                                                                         |                                                                                           |                                                                                           |
*2016 NJSLS:*

**RST:** Reading in Science and Technical Subjects

**WHST:** Writing in History, Science, and Technical Subjects

**SL:** Speaking and Listening

**L:** Language

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## Essential Question:

How are derivatives used to locate the extrema and points of inflection of a function?

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### Skills and Topics:

- relate knowledge of continuity to differentiability on an interval
- use the first derivative to locate minimum and maximum values of a function
- use the second derivative to determine concavity and to locate points of inflection of a function
- sketch graphs of functions based on shape and tendency
- determine answers and conclusions using a graphing calculator
- define “indeterminate forms” and provide examples
- apply limits and L’Hospital’s rule to aid in sketching graphs
- formulate a process for solving optimization problems
- connect concepts of minimums and maximums to solving problems related to geometry, economics, and banking
- justify solutions as logical or illogical
- find indefinite integrals (e.g., anti-derivatives)
- recognize and interpret integral notation
- approximate the area under a curve using Reimann Sums
- develop a definition of definite integrals using Reimann Sums
- state and apply the fundamental theorem of calculus
- use the properties of integrals to aid in problem solving
- determine the average value of a function and relate it to the mean value theorem
- apply the power rule and “u-substitution” to integrate functions
- use integration-by-parts to integrate functions
- improve the method of approximating the area under a curve using figures other than rectangles
- apply the trapezoid rule to approximate a definite integral linear approximations of \( f \)
- apply the Simpson’s rule to approximate a definite integral using parabolic approximations of \( f \)
- calculate the area between curves using horizontal and vertical slicing
- categorize integrals based on which type of slicing is most effective
- find the volume of a solid of revolution using the disk, washer, and shell methods
- predict the functions that form the boundaries of solids of revolutions
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<td>• formulate and debate hypotheses concerning the existence of extrema and points of inflection and their location relative to one another on the graph of the function</td>
<td>• model the vector qualities of position, velocity, and acceleration by analyzing the positive or negative area under a curve</td>
<td>• apply integration techniques to solving problems involving work, pressure, center of mass, consumer surplus, and arc length</td>
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Calculus
Course Requirements

Grade: 12  Prerequisite: Pre-Calculus  Credit Value: 5
Length of Course: Academic Year

Course Description

Calculus is designed for the student who will be taking calculus in college, but is not necessarily seeking advanced placement credit. The course builds upon concepts from Pre-Calculus, such as inverse functions, exponential functions, logarithms, and limits, and applies them to a detailed study of differentiation and integration. Considerable time is spent on the applications of calculus to the fields of business, finance, and physics through the study of related rates, optimization, and analysis of curves and slopes. Students are also required to justify answers and conclusions using graphing calculators or Microsoft Excel spreadsheets and then effectively communicate their reasoning orally and in writing. Calculus provides students with the conceptual framework and analytical skills necessary for application to real-world problems and to be successful in the college calculus course.

Course Content

This course will consist of the following units of study:

- Properties of Functions and Their Behavior
- Limits of Functions
- Derivatives
- Derivatives of Exponential and Logarithmic Functions
- Implicit Differentiation and Related Rates
- Graphing Functions Using Derivatives
- More with Graphing and Applications of The Derivative
- Indefinite and Definite Integrals
- Methods of Integration
- Solids of Revolution

Course Objectives

The student will demonstrate the ability to answer in detail the following essential questions:

- How do the characteristics of functions connect to the traits of their graphs?
- How do limits serve as a means to better understand functions and their behavior?
- How is the concept of derivative representative of instantaneous rate of change?
- How do exponential functions and logarithms apply to growth and decay?
Course Objectives (continued)

- When is implicit differentiation necessary?
- How are derivatives used to locate the extrema and points of inflection of a function?
- How does one use the derivative to solve optimization problems?
- What is the fundamental theorem of calculus and what is its significance?
- When is it necessary to integrate by parts?
- What are the similarities and differences among the disk, washer, and shell methods of integration?
- What are the post-graduation and/or career options that apply to the course content?

Evaluation Process

A final average of 65% or better is required to be awarded course credit. Throughout the length of this course, students may be evaluated on the basis of, but not limited to:

- Formative Assessments, such as writing prompts, journals, and portfolios
- Summative Assessments, such as quizzes, tests, and midterm and final examinations
- Performance Assessments, such as projects and presentations
- Technology-based Applications, such as electronic portfolios, Web Quests, ThinkQuest, and podcasting
- Class Participation
- Homework

Specific weights will be determined by course and level.
Calculus
Student Agreement

STUDENT NAME: ____________________________

Last Name ____________________________
First Name ____________________________

GRADE: ____________________________

My signature below indicates that I have received a copy of the Somerville Public Schools Course Requirements for Calculus.

I acknowledge my responsibility to read and understand all of the information contained in the Calculus Course Requirements information and syllabus packet.

__________________________________________________________________________

Student Signature ____________________________ Date ____________________________

Note: Please share the course requirements for Calculus with your parents.